Reasoning with Multilevel Contexts in Semantic Metanetwork

The goal of this topic is to study formal way to represent knowledge within several levels of contexts which can also been interpreted as levels of experts' competence.

Reference: Terziyan V., Puuronen S., Multilevel Context Representation Using Semantic Metanetwork, In: *Context-97 - International and Interdisciplinary Conference on Modeling and Using Context*, Rio de Janeiro, Brazil, Febr. 4-6, 1997, pp. 21-32.

A ***multilevel semantic network*** is proposed to be used to represent knowledge within several levels of ***contexts***. The zero level of representation is semantic network that includes knowledge about basic domain objects and their relations. The first level of presentation uses semantic network to represent contexts and their relationships. The second level presents relationships of ***metacontexts*** i.e. contexts of contexts, and so on at the higher levels. The topmost level includes knowledge which is considered to be "truth" in all the contexts. Such representation allows to ***reason with contexts*** towards solution of the following problems:

- to derive knowledge interpreted using all known levels of its context;
- to derive unknown knowledge when interpretation of it in some context and the context itself are known;
- to derive unknown knowledge about a context when it is known how the knowledge is interpreted in this context;
- to transform knowledge from one context to another.

A Semantic Metanetwork

Formally ***metanetwork*** is a quadruple **<*A,L,S,D*>**, where:

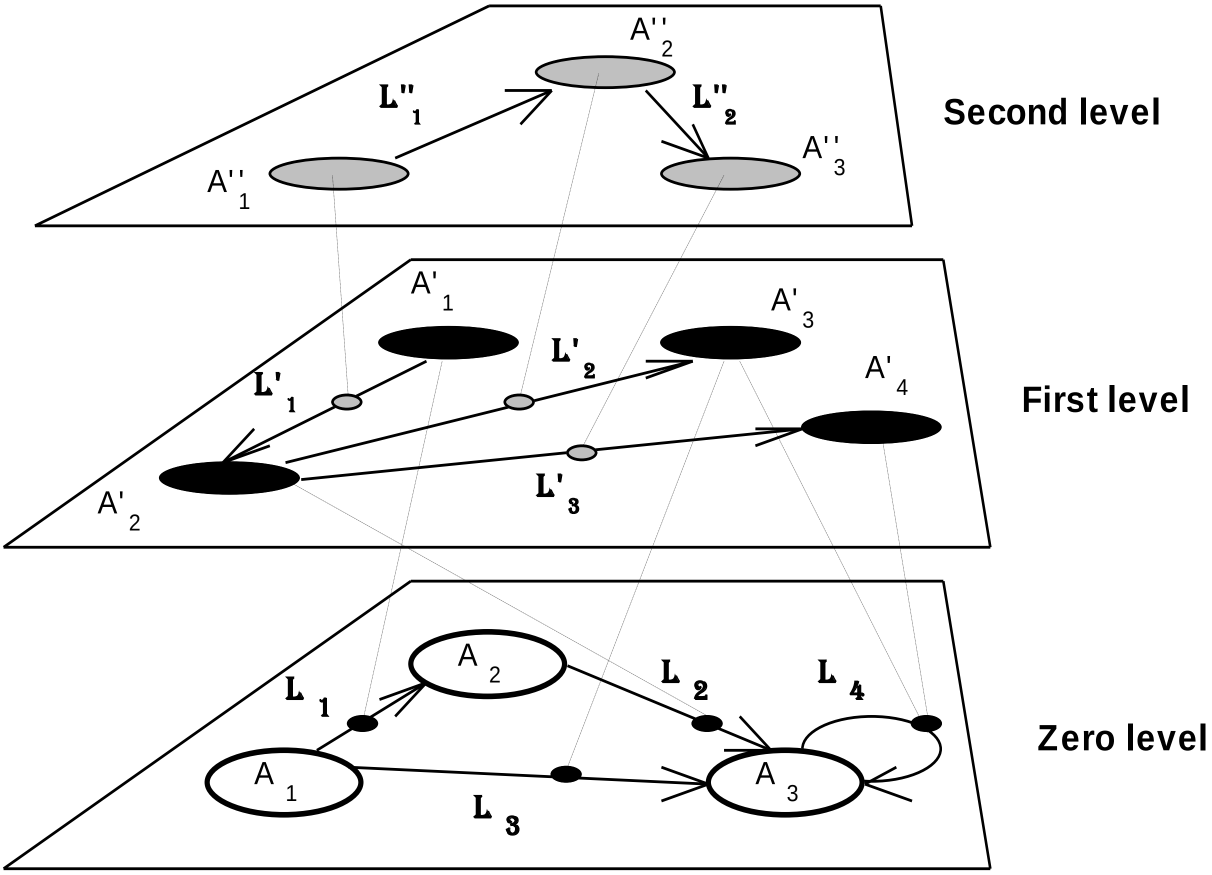
A is a set of objects which consists of the subset of basic domain objects $A_i^0, i = 1, \ldots, n_1$, and several subsets of contexts $A_i^{(d)}, i = 1, \ldots, n_d$, and $d = 1, \ldots, klev$ identifies the level of the metanetwork where the context appears;

L is a set of unique names of relations $L_k^{(d)}, k = 1, \ldots, m_d$ and $d = 0, \ldots, klev$ identifies the level of the metanetwork where the relation appears;

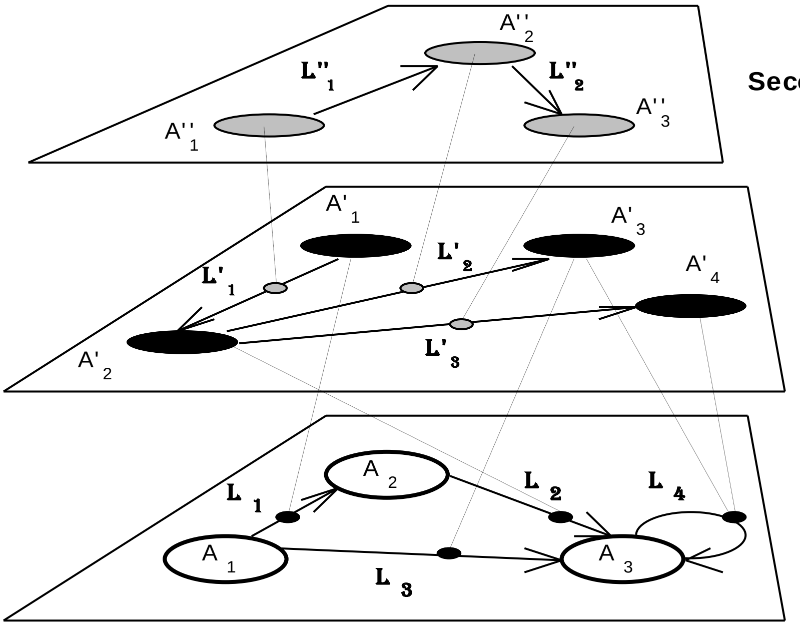
S is the set of relations $S_r^{(d)} = P(A_i^{(d)}, L_k^{(d)}, A_j^{(d)}), r = 1, \ldots, l_d$ composed in each level so that: $S^{(d)} = \bigwedge_r S_r^{(d)}$, and $P(A_i^{(d)}, L_k^{(d)}, A_j^{(d)})$ is true when there is the relation $L_k^{(d)} \in L$ between the objects $A_i^{(d)} and\ A_j^{(d)}, (A_i^{(d)}, A_j^{(d)} \in A)$ at the level d;

D is the set of context predicates $D_r^{(d)} = ist(A_i^{(d+1)}, S_r^{(d)})$ connecting contexts of the level $d+1$ to the relations of the level d and $ist(A_i^{(d+1)}, S_r^{(d)})$ is true if the relation $S_r^{(d)}$ holds in the context $A_i^{(d+1)}$.

A Semantic Metanetwork: An Example



A Semantic Metanetwork: An Example



$$A=\left\{A^{0},A^{'},A^{''}\right\},A^{0}=\left\{A_{1},A_{2},A_{3}\right\},A^{'}=\left\{A_{1}^{'},A_{2}^{'},A_{3}^{'},A_{4}^{'}\right\},A^{''}=\left\{A_{1}^{''},A_{2}^{''},A_{3}^{''}\right\}$$

are the ***objects***’ set and its three subsets accordingly to levels;

$$L=\left\{L^{0},L^{'},L^{''}\right\},L^{0}=\left\{L_{1},L_{2},L_{3},L_{4}\right\},L^{'}=\left\{L_{1}^{'},L_{2}^{'},L_{3}^{'}\right\},L^{''}=\left\{L_{1}^{''},L_{2}^{''}\right\}$$

are the set of ***relations’ names*** and its three subsets accordingly to levels;

$$S=\left\{S^{0},S^{'},S^{''}\right\},\ S^{0}=\left\{S_{1},S_{2},S_{3},S_{4}\right\},S^{'}=\left\{S_{1}^{'},S_{2}^{'},S_{3}^{'}\right\},S^{''}=\left\{S_{1}^{''},S_{2}^{''}\right\}$$

are the ***set of relations*** and its three subsets accordingly to levels;

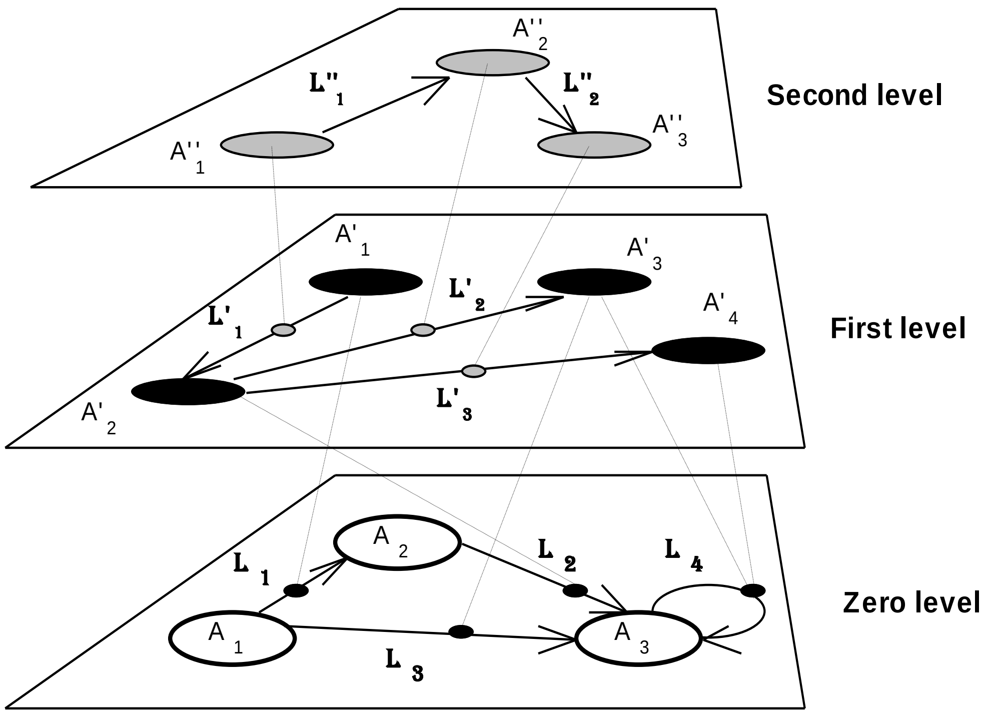
$$S_1=P(A_1,L_1,A_2),S_2=P(A_2,L_2,A_3),S_3=P(A_1,L_3,A_3),S_4=P(A_3,L_4,A_3);$$

$$S_1^{'}=P(A_1^{'},L_1^{'},A_2^{'}),S_2^{'}=P(A_2^{'},L_2^{'},A_3^{'}),S_3^{'}=P(A_2^{'},L_3^{'},A_4^{'});$$

$$S_1^{''}=P(A_1^{''},L_1^{''},A_2^{''}),S_2^{''}=P(A_2^{''},L_2^{''},A_3^{''})$$

are the ***relations*** of all the three levels in a ***predicate form***;

A Semantic Metanetwork: An Example



$$D = \left\{ D^0, D^{'} \right\},\ D^0 = \left\{ D_1, D_2, D_3, D_4, D_5 \right\}, D^{'} = \left\{ D_1^{'}, D_2^{'}, D_3^{'} \right\}$$

are the ***set of context predicates*** and its two subsets accordingly to levels;

$$D_1 = ist(A_1^{'}, S_1), D_2 = ist(A_2^{'}, S_2), D_3 = ist(A_3^{'}, S_3),$$

$$D_4 = ist(A_3^{'}, S_4), D_5 = ist(A_4^{'}, S_4)$$

are the ***context predicates*** describing context relationships between ***zero and first*** levels;

$$D_1^{'} = ist(A_1^{''}, S_1^{'}), D_2^{'} = ist(A_2^{''}, S_2^{'}), D_3^{'} = ist(A_3^{''}, S_3^{'})$$

are the ***context predicates*** describing context relationships between ***first and second*** levels.

Some Concepts

Further in formulas we will use:

"$\Leftrightarrow$" to mark the logical ***equivalence*** between any two predicates;

"$\Rightarrow$" to mark the logical ***consequence*** between two predicates;

"$\equiv$" between names of relations or objects to mark ***equality of semantic meanings*** of these two names;

"$\neq$" between names of relations to mark ***inequality of semantic meanings*** of these two names.

The ***main equation*** used in proofs:

$$(P(A_i, L_k, A_j) \Leftrightarrow P(A_i, L_m, A_j)) \Leftrightarrow L_k \equiv L_m$$

Semantic Constants: The Always True Relations - Relation of “Difference” and Relation of “Same”

We consider knowledge about the difference as semantic constant ***DIF*** which denotes the relation between every pair of different objects:

$$\forall A_i, A_{j(j \neq i)} (P(A_i, DIF, A_j) \Leftrightarrow true)$$

Semantic constant of equivalence ***SAME***:

$$\forall A_i (P(A_i, SAME, A_i) \Leftrightarrow true)$$

The last formula means that if there is not any knowledge about properties of some object, than at least one property always holds - to be same to itself.

Semantic Constants: The Always False Relation and Relation of "Similarity"

We define ***NIL*** relation as relation which ***never holds*** between any two objects or among properties of any object by the following way:

$$\forall A_i, A_{j(j \neq i)} (\neg P(A_i, DIF, A_j) \Leftrightarrow P(A_i, NIL, A_j) \Leftrightarrow false)$$

$$\forall A_i (\neg P(A_i, SAME, A_i) \Leftrightarrow P(A_i, NIL, A_i) \Leftrightarrow false)$$

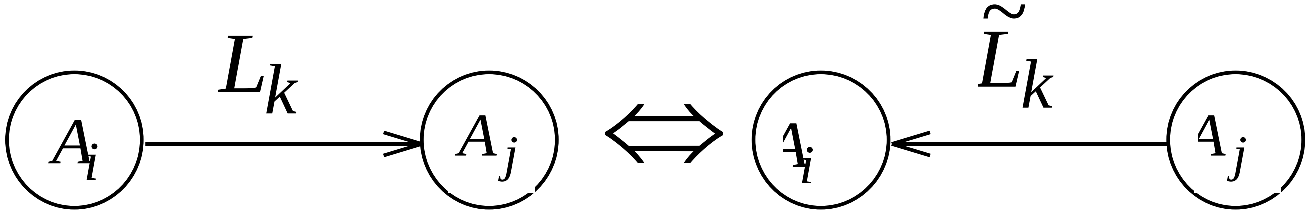
The relation of ***"similarity"***:

$$(P(A_s, L_k, A_i) \Leftrightarrow P(A_s, L_k, A_j), \forall A_s \in A, \forall L_k \in L) \Leftrightarrow$$

$$\Leftrightarrow P(A_i, SAME^b, A_j)$$

where $\boldsymbol{SAME^b}$ means the binary equivalent of ***SAME*** relation. This relation does not mean the negation of ***DIF*** relation, because it means only that two different objects have the same semantics.

Semantic operations: Semantic inversion



$$P(A_i, L_k, A_j) \Leftrightarrow P(A_j, \tilde{L}_k, A_i)$$

where $\tilde{L}_k$ is the new inverse relation. It is named with unique symbol L_m and added to the set $\boldsymbol{L}$.

For example, if L_k = <*to_punish*>, then L_m = <*to_be_punished*> and $L_m \equiv \tilde{L}_k$.

Similarly, let L_n = <*to_be_on_the_left_side_of*>, then L_q = <*to_be_on_the_right_side_of*> and $L_q \equiv \tilde{L}_n$.

In the case when L_k is a *property of an object*:

$$P(A_i, L_k, A_i) \Leftrightarrow P(A_i, \tilde{L}_k, A_i) \text{ and } L_k \equiv \tilde{L}_k .$$

The obvious property for the semantic inversion operation is *double inversion*: $\tilde{\tilde{L}}_k \equiv L_k$.

Semantic Operations: Semantic Negation

The ***semantic negation*** operation means changing the name of a relation if the value of appropriate predicate is false:

$$\neg P(A_i, L_k, A_j) \Leftrightarrow P(A_i, \overline{L}_k, A_j)$$

where $\overline{L}_k$ is the new relation. It can be named with unique symbol L_m and added to the set $\boldsymbol{L}$. If, for example:

P(<Mary>, <to_love>, <Tom>) = false,

then it means the same as:

P(<Mary>, <not_to_love>, <Tom>) = true.

Thus, if L_k = *<to_love>* and L_m= *<not_to_love>*, then $L_m \equiv \overline{L}_k$.

Properties of Semantic Negation

- double negation: $\overline{\overline{L}}_k \equiv L_k$;

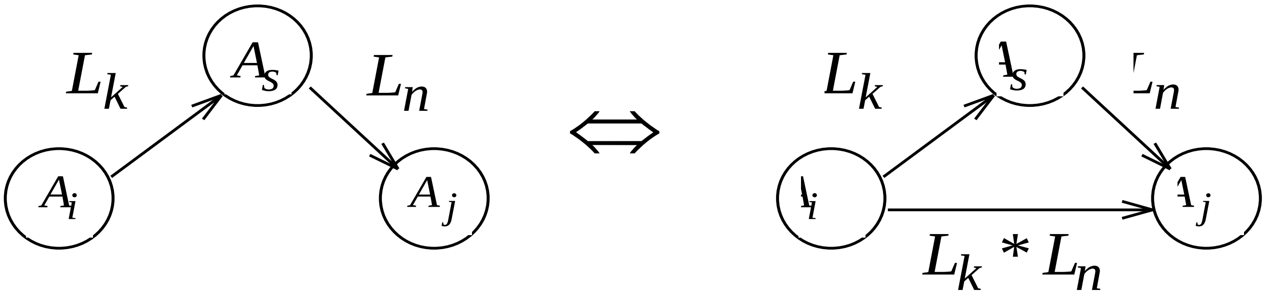
- negation of inversion: $\overline{\tilde{L}}_k \equiv \tilde{\overline{L}}_k$;

  Proof:

$$(P(A_i, \overline{\tilde{L}}_k, A_j) \Leftrightarrow \neg P(A_i, \tilde{L}_k, A_j) \Leftrightarrow \neg P(A_j, L_k, A_i) \Leftrightarrow$$

$$\Leftrightarrow P(A_j, \overline{L}_k, A_i) \Leftrightarrow P(A_i, \tilde{\overline{L}}_k, A_j)) \Leftrightarrow \overline{\tilde{L}}_k \equiv \tilde{\overline{L}}_k$$

Semantic Operations: Semantic Multiplication (Composition)



$$P(A_i, L_k, A_s) \wedge P(A_s, L_n, A_j) \Leftrightarrow P(A_i, L_k * L_n, A_j),$$

where $L_k * L_n$ is the new relation. It can be named with unique symbol L_m and added to the set ***L***. If, for example, it is true that:

P(<Mary>, <to_be_married_with>, <Tom>) and

P(<Tom>, <to_have_mother>, <Diana>),

then it is also true that:

P(<Mary>, <to_have_mother-in-law>, <Diana>).

Thus, if:

L_k = *<to_be_married_with>*, L_n = *<to_have_mother>*,

and L_m = *<to_have_mother-in-law>*, then $L_m \equiv L_k * L_n$.

Properties of Semantic Multiplication:

- non-commutativity :

$$\neg(L_k * L_n \equiv L_n * L_k),$$

$$\forall L_k, L_n (L_k \neq L_n \neq DIF \neq SAME^b)$$

- transitivity:

$$L_k * (L_n * L_m) \equiv (L_k * L_n) * L_m; \quad \textit{Proof:}$$

$$(P(A_i, L_k * (L_n * L_m), A_j) \Leftrightarrow$$

$$\Leftrightarrow P(A_i, L_k, A_s) \wedge P(A_s, L_n * L_m, A_j) \Leftrightarrow$$

$$\Leftrightarrow P(A_i, L_k, A_s) \wedge P(A_s, L_n, A_t) \wedge P(A_t, L_m, A_j) \Leftrightarrow$$

$$\Leftrightarrow P(A_i, L_k * L_n, A_t) \wedge P(A_t, L_m, A_j) \Leftrightarrow$$

$$\Leftrightarrow P(A_i, (L_k * L_n) * L_m, A_j)) \Leftrightarrow$$

$$\Leftrightarrow L_k * (L_n * L_m) \equiv (L_k * L_n) * L_m$$

- inversion over multiplication:

$$\sim(L_k * L_n) \equiv \tilde{L}_n * \tilde{L}_k; \qquad \textit{Proof:}$$

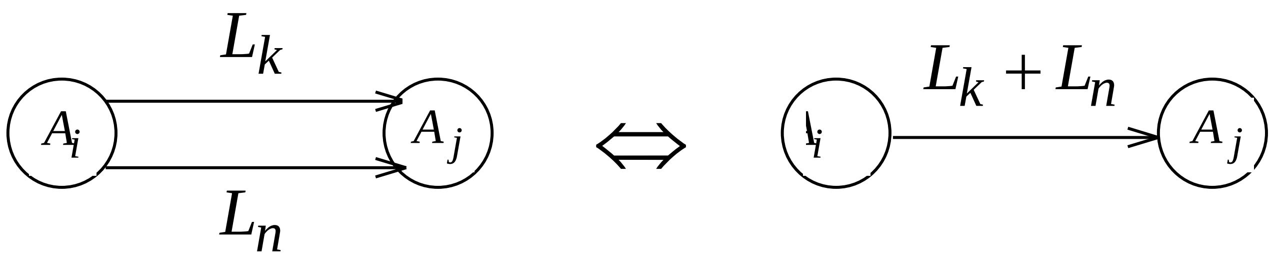
$$(P(A_j, L_k * L_n, A_i) \Leftrightarrow P(A_j, L_k, A_s) \wedge P(A_s, L_n, A_i) \Leftrightarrow$$

$$\Leftrightarrow P(A_s, \tilde{L}_k, A_j) \wedge P(A_i, \tilde{L}_n, A_s) \Leftrightarrow$$

$$\Leftrightarrow P(A_i, \tilde{L}_n, A_s) \wedge P(A_s, \tilde{L}_k, A_j) \Leftrightarrow$$

$$\Leftrightarrow P(A_i, \tilde{L}_n * \tilde{L}_k, A_j)) \Leftrightarrow \sim(L_k * L_n) \equiv \tilde{L}_n * \tilde{L}_k.$$

Semantic Operations: Semantic Addition



$$P(A_i, L_k, A_j) \wedge P(A_i, L_n, A_j) \Leftrightarrow P(A_i, L_k + L_n, A_j),$$

where $L_k + L_n$ is the new relation. It can be named with unique symbol L_m and added to the set $\boldsymbol{L}$. If, for example, it is true that:

P(<Mary>, <to_give_birth_to>, <Tom>) and

P(<Mary>, <to_take_care_of>, <Tom>), then:

P(<Mary>, <to_be_mother_of>, <Tom>) is also true.

Thus, if:

L_k *= <to_give_birth_to>,* L_n *= <to_take_care_of>,* and

$L_m = L_k + L_n$ *= <to_be_mother_of>.*

It is also possible to ***sum properties***. If, for example, it is true that *P(<Tom>, <to_be_clever>, <Tom>)* and *P(<Tom>, <to_be_rich>,<Tom>),* then it is also true that: *P(<Tom>, <to_be_clever_and_rich>, <Tom>).*

Properties of Semantic Addition

- commutativity: $L_k + L_n \equiv L_n + L_k$;
- transitivity: $L_k + (L_n + L_m) \equiv (L_k + L_n) + L_m$;
- reflexivity: $L_k + L_k \equiv L_k$;
- inversion over sum: $\sim(L_k + L_n) \equiv \tilde{L}_k + \tilde{L}_n$;
- distributivity (left):

$L_k * (L_m + L_n) \equiv L_k * L_m + L_k * L_n$; *Proof:*

$$(P(A_i, L_k * (L_m + L_n), A_j) \Leftrightarrow P(A_i, L_k, A_s) \wedge P(A_s, L_m + L_n, A_j) \Leftrightarrow$$
$$\Leftrightarrow P(A_i, L_k, A_s) \wedge P(A_s, L_m, A_j) \wedge P(A_s, L_n, A_j) \Leftrightarrow$$
$$\Leftrightarrow P(A_i, L_k, A_s) \wedge P(A_i, L_k, A_s) \wedge$$
$$\wedge P(A_s, L_m, A_j) \wedge P(A_s, L_n, A_j) \Leftrightarrow (P(A_i, L_k, A_s) \wedge P(A_s, L_m, A_j)) \wedge$$
$$\wedge (P(A_i, L_k, A_s) \wedge P(A_s, L_n, A_j)) \Leftrightarrow$$
$$\Leftrightarrow P(A_i, L_k * L_m, A_j) \wedge P(A_i, L_k * L_n, A_j) \Leftrightarrow$$
$$\Leftrightarrow P(A_i, L_k * L_m + L_k * L_n, A_j)) \Leftrightarrow$$
$$\Leftrightarrow L_k * (L_m + L_n) \equiv L_k * L_m + L_k * L_n;$$

- distributivity (right)

$$(L_k + L_m) * L_n \equiv L_k * L_n + L_m * L_n .$$

Semantic Closeness

We define the ***L***-set $L\colon\left\{L_1, L_2, \dots L_n\right\}$ as ***semantically closed*** set of relations' names if the result of any semantic operation with any operands from the ***L***-set also belongs to the ***L*** -set:

$$\tilde{L}_k \in L, \forall L_k \in L; \quad \overline{L}_k \in L, \forall L_k \in L;$$

$$L_k * L_m \in L, \forall L_k, L_m \in L;$$

$$L_k + L_m \in L, \forall L_k, L_m \in L;$$

$$\tilde{L}\colon\left\{\tilde{L}_1, \tilde{L}_2, \dots, \tilde{L}_n\right\} \equiv L; \quad \overline{L}\colon\left\{\overline{L}_1, \overline{L}_2, \dots \overline{L}_n\right\} \equiv L;$$

$$(L_i * L)\colon\left\{L_i * L_1, L_i * L_2, \dots, L_i * L_n\right\} \equiv L, \forall L_i \in L;$$

$$(L * L_i)\colon\left\{L_1 * L_i, L_2 * L_i, \dots, L_n * L_i\right\} \equiv L, \forall L_i \in L;$$

$$(L_i + L)\colon\left\{L_i + L_1, L_i + L_2, \dots, L_i + L_n\right\} \equiv L, \forall L_i \in L.$$

Properties of Semantic Constants

The negation operation defines the main relationship between the two main constants as follows:

$$\overline{DIF} \equiv \overline{SAME} \equiv NIL$$

***Properties of DIF*:**

- neutrality to semantic sum: $DIF + L_k \equiv L_k$.

Proof:

$$(P(A_i, DIF + L_k, A_j) \Leftrightarrow \underbrace{P(A_i, DIF, A_j)}_{true} \wedge P(A_i, L_k, A_j) \Leftrightarrow$$
$$\Leftrightarrow P(A_i, L_k, A_j)) \Leftrightarrow DIF + L_k \equiv L_k \quad ;$$

- elimination in semantic multiplication:

$$DIF * L_k \equiv L_k * DIF \equiv DIF \; ;$$

- inversion of ***DIF***: $\sim(DIF) \equiv DIF$.

Properties of Semantic Constants

***Properties of SAME*:**

- inversion of *SAME* $\sim(SAME) \equiv SAME$;
- neutrality to semantic sum $SAME + L_k \equiv L_k$;

Properties of Semantic Constants

Properties of $SAME^b$:

- inversion of $SAME^b$: $\sim(SAME^b) \equiv SAME^b$;
- neutrality to semantic sum: $SAME^b + L_k \equiv L_k$;
- neutrality to semantic multiplication:

$$L_k * SAME^b \equiv SAME^b * L_k \equiv L_k;$$

- annihilation: a) $L_k + \tilde{L}_k \equiv \begin{cases} SAME^b, \textit{ for relations}; \\ L_k, \textit{ for properties.} \end{cases}$

  b) $L_k * \tilde{L}_k \equiv \tilde{L}_k * L_k \equiv SAME^b$.

- elimination (left): $L_k + L_k * L_m \equiv L_k * L_m$.

  Proof:

$$L_k + L_k * L_m \equiv L_k * SAME^b + L_k * L_m \equiv$$

$$\equiv L_k * (SAME^b + L_m) \equiv L_k * L_m \quad ;$$

- elimination (right): $L_k + L_m * L_k \equiv L_m * L_k$.

Semantic Equations

$$F_1(L^1)^{L_x} = F_2(L^2)^{L_x}$$

where $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ are functions, which use semantic operations and they have known operands respectively subsets $\boldsymbol{L^1}$, $\boldsymbol{L^2}$ of the $\boldsymbol{L}$-set and one ***unknown operand*** $\boldsymbol{L_x}$, which is defined on the whole $\boldsymbol{L}$-set.

We use "**=**" in semantic equations to tell the difference between ***semantic equations*** and ***semantic identities*** where we use "$\equiv$".

The ***solution*** of such equation is the relation $\boldsymbol{L_w}$, which, being substituted in an equation instead of the operand $\boldsymbol{L_x}$ will transform it to an identity. We will use notation $(equation)^{L_x = L_w}$ as a statement that relation $\boldsymbol{L_w}$ is the solution of the *equation*.

Semantic Equations: Basic Properties

$$(F_1(L^1) = F_2(L^2))^{L_x = L_w} \Leftrightarrow$$
$$\Leftrightarrow (\tilde{F}_1(L^1) = \tilde{F}_2(L^2))^{L_x = L_w}$$

$$(F_1(L^1) = F_2(L^2))^{L_x = L_w} \Leftrightarrow$$
$$\Leftrightarrow (\overline{F}_1(L^1) = \overline{F}_2(L^2))^{L_x = L_w}$$

$$(F_1(L^1) = F_2(L^2))^{L_x = L_w} \Leftrightarrow$$
$$\Leftrightarrow (F_1(L^1) + L_k = F_2(L^2) + L_k)^{L_x = L_w}, \forall L_k \in L$$

$$(F_1(L^1) = F_2(L^2))^{L_x = L_w} \Leftrightarrow$$
$$\Leftrightarrow (F_1(L^1) * L_k = F_2(L^2) * L_k)^{L_x = L_w}, \forall (L_k \neq DIF \in L)$$

$$(F_1(L^1) = F_2(L^2))^{L_x = L_w} \Leftrightarrow$$
$$\Leftrightarrow (L_k * F_1(L^1) = L_k * F_2(L^2))^{L_x = L_w}, \forall (L_k \neq DIF \in L)$$

Semantic Equations: Solution of Basic Equations

$$\widetilde{L}_x = L_i \Rightarrow \widetilde{\widetilde{L}}_x = \widetilde{L}_i \Rightarrow L_x = \widetilde{L}_i\ ;$$

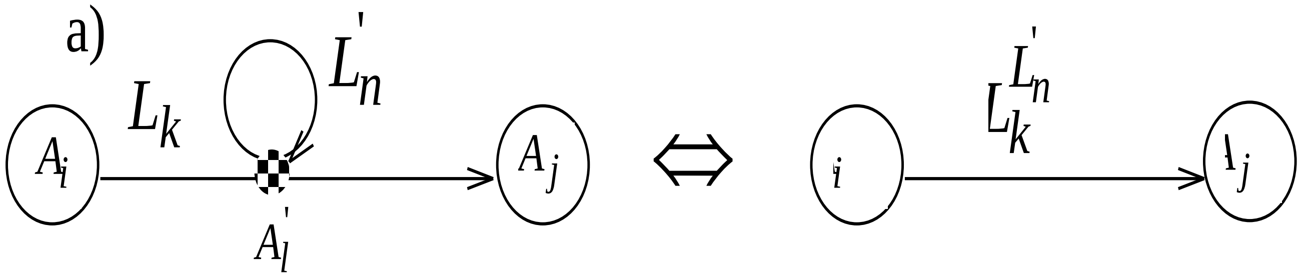
$$\overline{L}_x = L_i \Rightarrow \overline{\overline{L}}_x = \overline{L}_i \Rightarrow L_x = \overline{L}_i\ ;$$

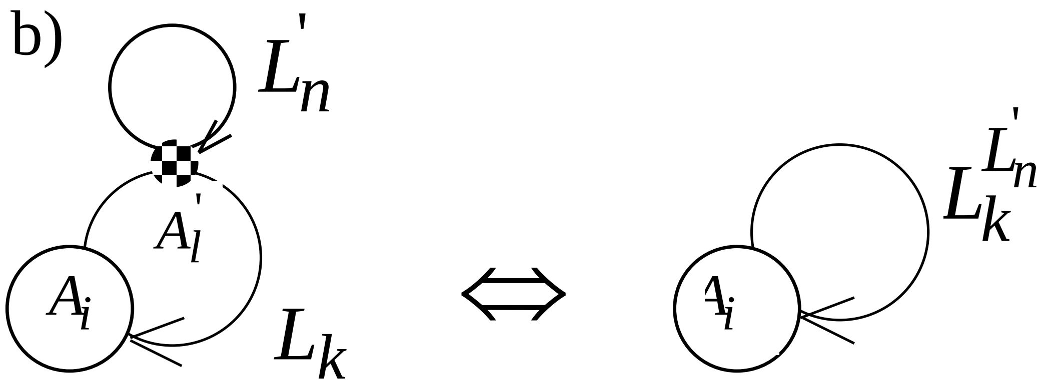
$$L_x + L_i = L_j \Rightarrow L_x + L_i + \widetilde{L}_i = L_j + \widetilde{L}_i \Rightarrow$$
$$\Rightarrow L_x + SAME^b = L_j + \widetilde{L}_i \Rightarrow L_x = L_j + \widetilde{L}_i\ ;$$

$$L_x * L_i = L_j \Rightarrow L_x * L_i * \widetilde{L}_i = L_j * \widetilde{L}_i \Rightarrow$$
$$\Rightarrow L_x * SAME^b = L_j * \widetilde{L}_i \Rightarrow L_x = L_j * \widetilde{L}_i\ ;$$

$$L_i * L_x = L_j \Rightarrow \widetilde{L}_i * L_i * L_x = \widetilde{L}_i * L_j \Rightarrow$$
$$\Rightarrow SAME^b * L_x = \widetilde{L}_i * L_j \Rightarrow L_x = \widetilde{L}_i * L_j\ .$$

Operation of Semantic Interpretation





$$P(A_i, L_k, A_j) \wedge P(A'_l, L'_n, A'_l) \wedge ist(A'_l, P(A_i, L_k, A_j)) \Leftrightarrow$$

$$\Leftrightarrow P(A_i, L_k^{L'_n}, A_j)$$

where $L_k^{L'_n}$ denotes the interpretation of the knowledge L_k using knowledge $L'n$ about the context A'_l. This can be named and included to the set $\boldsymbol{L}$.

Expansion of Definitions for Semantic Constants:

1. $L_k^{SAME} \equiv L_k$.

This means an abstract case when the context of a relation contains only the universal property ***SAME***. In such case the context cannot change an interpretation of this relation;

2. $SAME^{L_k} \equiv L_k$.

This means also an abstract case when the knowledge being interpreted contains only the universal property ***SAME***. In such case the only knowledge obtained as a result of interpretation is the knowledge about context;

3. $(SAME^b)^{L_k} \equiv L_k^b$.

This means that interpretation of the equivalence relation between two objects inherits the properties of context L_k producing its binary equivalent L_k^b ;

Expansion of Definitions for Semantic Constants:

$$4.\ (L_k^{L_m} \equiv L_n) \Leftrightarrow (L_n^{\overline{L}_m} \equiv L_k)$$

- *axiom of extracting knowledge*.

This means that if some knowledge $\boldsymbol{L_n}$ has been obtained as interaction of knowledge $\boldsymbol{L_k}$ and property $\boldsymbol{L_m}$ of some context, then removal of that property from the context of the interpretation result leads to the restoration of initial knowledge;

Consequence: $L_k^{\overline{L}_k} \equiv SAME^{[b]}$;

$$5.\ (L_k^{L_m} \equiv L_n) \Leftrightarrow (\overline{L}_k^{L_n} \equiv L_m)$$

- *axiom of extracting context*.

This means that if some knowledge $\boldsymbol{L_n}$ has been obtained as interaction of knowledge $\boldsymbol{L_k}$ and property $\boldsymbol{L_m}$ of some context, then removal of the initial knowledge $\boldsymbol{L_k}$ in the context of the interpretation result leads to the restoration of initial context.

Consequence: $\overline{L}_k^{L_k} \equiv SAME^{[b]}$.

Transformations with contexts in the Algebra

Inversion in the context.

The inverse result of interpretation a relation in a context is equal to the result of interpretation the inverse relation in the same context. Formally:

$$\sim\left(L_k^{L_m}\right) \equiv \tilde{L}_k^{L_m}\ .$$

Negation in the context.

The negative result of interpretation a relation in a context is equal to the result of interpretation the negative relation in the same context, and also it equals to the result of interpretation of this relation in a negative context. Formally:

$$\overline{L_k^{L_m}} \equiv \overline{L}_k^{L_m} \equiv L_k^{\overline{L_m}}\ .$$

Transformations with contexts in the Algebra

Addition in the context.

The result of interpretation of the sum of two not-conflicting relations in a context is equal to the semantic sum of these two relations interpreted separately in the same context. Formally:

$$(L_k + L_m)^{L_n} \equiv L_k^{L_n} + L_m^{L_n}, \quad when\ L_k \neq \overline{L}_m.$$

Multiplication in the context.

The result of interpretation of the semantic multiplication of two not-conflicting relations in a context is equal to the semantic multiplication of these two relations interpreted separately in the same context. Formally:

$$(L_k * L_m)^{L_n} \equiv L_k^{L_n} * L_m^{L_n}, \quad when\ L_k \neq \overline{L}_m.$$

Transformations with contexts in the Algebra

Interpretation in the sum of contexts.

The result of interpretation of a relation in a semantic sum of two not-conflicting contexts is equal to the semantic sum of this relation interpreted separately in these two contexts. Formally:

$$L_k^{L_m+L_n} \equiv L_k^{L_m} + L_k^{L_n}, \;\; when\; L_m \neq \overline{L}_n \,.$$

Addition of interpreted relations.

The result of a semantic sum of two not-conflicting relations interpreted separately in two not-conflicting contexts is equal to the semantic sum of these relations interpreted in the semantic sum of these contexts:

$$L_k^{L_m} + L_r^{L_n} \Rightarrow (L_k + L_r)^{L_m+L_n}, \;\; when\; L_k \neq \overline{L}_r,\; L_m \neq \overline{L}_n \,.$$

Transformations with contexts in the Algebra

Multiplication of interpreted relations.

The result of a semantic multiplication of two not-conflicting relations interpreted separately in two not-conflicting contexts is equal to the semantic multiplication of these relations interpreted in the semantic sum of these contexts:

$$L_k^{L_m} * L_r^{L_n} \Rightarrow (L_k * L_r)^{L_m + L_n}, \ \ when\ L_k \neq \overline{L}_r,\ L_m \neq \overline{L}_n.$$

Multilevel interpretation.

The result of a relation interpretation in several levels of contexts does not depend on the order of interpretation:

$$(L_k^{L_m})^{L_n} \equiv L_k^{(L_m^{L_n})}, \ \ when\ L_m \neq \overline{L}_k,\ L_m \neq \overline{L}_n.$$

Equations with Contexts

Basic rules:

$$(F_1(L^1) = F_2(L^2))^{L_x = L_w} \Leftrightarrow$$

$$\Leftrightarrow (F_1(L^1)^{L_k} = F_2(L^2)^{L_k})^{L_x = L_w}, \forall L_k \in L$$

$$(F_1(L^1) = F_2(L^2))^{L_x = L_w} \Leftrightarrow$$

$$\Leftrightarrow (L_k^{F_1(L^1)} = L_k^{F_2(L^2)})^{L_x = L_w}, \forall L_k \in L$$

Solution of basic equations:

$$L_x^{L_i} = L_j \Leftrightarrow (L_x^{L_i})^{\overline{L_i}} = L_j^{\overline{L_i}} \Leftrightarrow$$

$$\Leftrightarrow L_x^{(L_i{}^{\overline{L_i}})} = L_j^{\overline{L_i}} \Leftrightarrow L_x^{SAME} = L_j^{\overline{L_i}} \Leftrightarrow L_x = L_j^{\overline{L_i}};$$

$$L_i^{L_x} = L_j \Leftrightarrow \overline{L_i}^{(L_i^{L_x})} = \overline{L_i}^{L_j} \Leftrightarrow$$

$$\Leftrightarrow (\overline{L_i}^{L_i})^{L_x} = \overline{L_i}^{L_j} \Leftrightarrow SAME^{L_x} = \overline{L_i}^{L_j} \Leftrightarrow L_x = \overline{L_i}^{L_j}.$$

Semantic Equations: An Example

$$\tilde{L}_1 * L_2^{(\tilde{L}_3 * \tilde{L}_x * L_4 + L_5)^{\tilde{L}_6}} * L_7 + L_8 = L_9 \Leftrightarrow$$

$$\Leftrightarrow \tilde{L}_1 * L_2^{(\tilde{L}_3 * \tilde{L}_x * L_4 + L_5)^{\tilde{L}_6}} * L_7 = L_9 + \tilde{L}_8 \Leftrightarrow$$

$$\Leftrightarrow L_2^{(\tilde{L}_3 * \tilde{L}_x * L_4 + L_5)^{\tilde{L}_6}} * L_7 = \tilde{\tilde{L}}_1 * (L_9 + \tilde{L}_8) \Leftrightarrow$$

$$\Leftrightarrow L_2^{(\tilde{L}_3 * \tilde{L}_x * L_4 + L_5)^{\tilde{L}_6}} = L_1 * (L_9 + \tilde{L}_8) * \tilde{L}_7 \Leftrightarrow$$

$$\Leftrightarrow L_2^{(\tilde{L}_3 * \tilde{L}_x * L_4 + L_5)} = (L_1 * (L_9 + \tilde{L}_8) * \tilde{L}_7)^{\bar{\tilde{L}}_6} \Leftrightarrow$$

$$\Leftrightarrow \tilde{L}_3 * \tilde{L}_x * L_4 + L_5 = \bar{L}_2^{(L_1 * (L_9 + \tilde{L}_8) * \tilde{L}_7)^{\bar{\tilde{L}}_6}} \Leftrightarrow$$

$$\Leftrightarrow \tilde{L}_3 * \tilde{L}_x * L_4 = \bar{L}_2^{(L_1 * (L_9 + \tilde{L}_8) * \tilde{L}_7)^{\bar{\tilde{L}}_6}} + \tilde{L}_5 \Leftrightarrow$$

$$\Leftrightarrow \tilde{L}_x * L_4 = \tilde{\tilde{L}}_3 * (\bar{L}_2^{(L_1 * (L_9 + \tilde{L}_8) * \tilde{L}_7)^{\bar{\tilde{L}}_6}} + \tilde{L}_5) \Leftrightarrow$$

$$\Leftrightarrow \tilde{L}_x = L_3 * (\bar{L}_2^{(L_1 * (L_9 + \tilde{L}_8) * \tilde{L}_7)^{\bar{\tilde{L}}_6}} + \tilde{L}_5) * \tilde{L}_4 \Leftrightarrow$$

$$\Leftrightarrow L_x = \sim(L_3 * (\bar{L}_2^{(L_1 * (L_9 + \tilde{L}_8) * \tilde{L}_7)^{\bar{\tilde{L}}_6}} + \tilde{L}_5) * \tilde{L}_4).$$

Reasoning with Contexts

Deriving an interpreted knowledge (decontextualization).

Deriving of interpreted knowledge means here deriving the formula for knowledge interpreted using all known levels of its context.

We define the ***knowledge of a relation*** $L_{A_i - A_j}$ between any pairs of objects (A_j, A_k) from the same level of a semantic metanetwork as a semantic sum over all possible paths between these objects (A_j, A_k) that exist at this level of the metanetwork.

We define the ***knowledge*** L_{A_i} ***of an object*** A_i of a semantic metanetwork in one level as a semantic sum over all knowledge of the relations that connect this object with all objects of the same level, including the object itself:

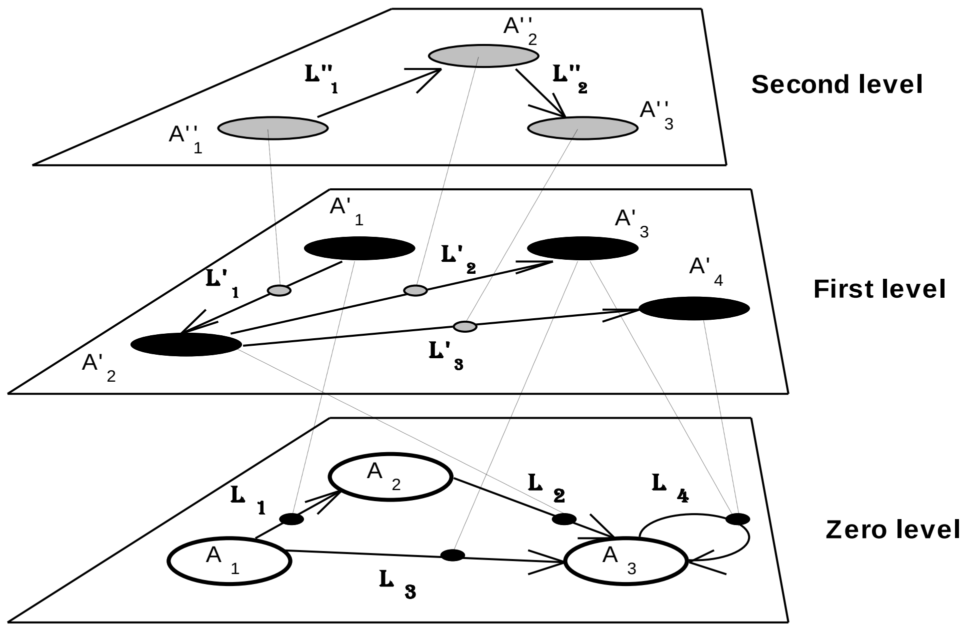
$$L_{A_i} = \sum_j L_{A_i - A_j} .$$

The ***interpreted knowledge*** of any relation, considering all contexts and metacontexts, is derived by the following schema:

$$< \text{interpreted knowledge} > \; =$$

$$= \; < \text{knowledge} >^{<\text{knowl. about context}>^{\cdots^{<\text{knowl. about metacont. of n-th level}>}}} .$$

Example of Decontextualization (1)



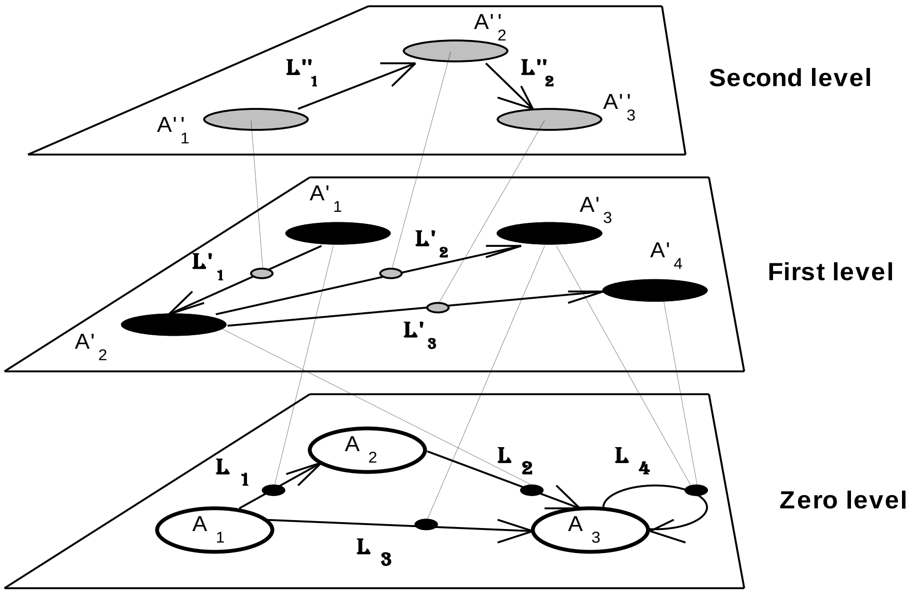
Let us derive the interpreted knowledge of the relation between $\boldsymbol{A_1}$ and $\boldsymbol{A_3}$. We start from the top level of the metanetwork and define knowledge about metacontexts A_1'', A_2'', A_3'':

$$L_{A_1''} = L_{A_1''-A_2''} + L_{A_1''-A_3''} + L_{A_1''-A_1''} =$$

$$= L_1'' + L_1'' * L_2'' + SAME = L_1'' * (SAME + L_2'') = L_1'' * L_2'';$$

$$L_{A_2''} = \tilde{L}_1'' + L_2''; \qquad L_{A_3''} = \tilde{L}_2'' * \tilde{L}_1''.$$

Example of Decontextualization (2)



We continue at the first level and derive the interpreted knowledge of the first level relations:

$$L_{A'_1-A'_2} = (L'_1)^{L_{A''_1}} = (L'_1)^{L''_1 * L''_2};$$

$$L_{A'_1-A'_3} = (L'_1)^{L_{A''_1}} * (L'_2)^{L_{A''_2}} =$$

$$= (L'_1)^{L''_1 * L''_2} * (L'_2)^{\tilde{L}''_1 + L''_2} = (L'_1 * L'_2)^{L''_1 * L''_2 + \tilde{L}''_1};$$

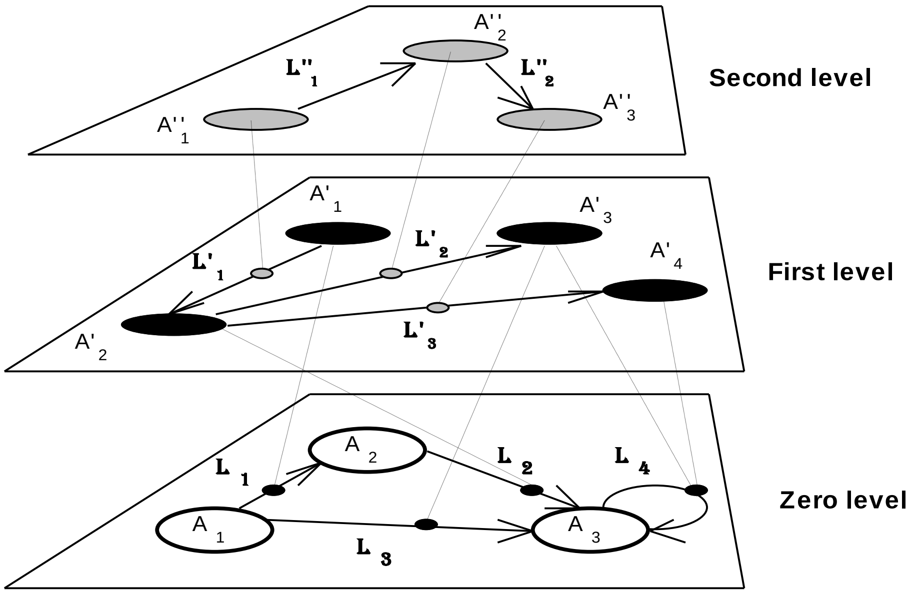
$$L_{A'_1-A'_4} = (L'_1)^{L_{A''_1}} * (L'_3)^{L_{A''_3}} = (L'_1)^{L''_1 * L''_2} * (L'_3)^{\tilde{L}''_2 * \tilde{L}''_1};$$

...

$$L_{A'_4-A'_3} = (\tilde{L}'_3)^{L_{A''_3}} * (L'_2)^{L_{A''_2}} =$$

$$= (\tilde{L}'_3)^{\tilde{L}''_2 * \tilde{L}''_1} * (L'_2)^{\tilde{L}''_1 + L''_2} = (\tilde{L}'_3 * L'_2)^{\tilde{L}''_2 * \tilde{L}''_1 + L''_2}.$$

Example of Decontextualization (3)



The knowledge about contexts A_1', A_2', A_3' of the first level is derived as follows:

$$L_{A_1'} = (L_1' * L_2')^{L_1'' * L_2'' + \tilde{L}_1''} + (L_1' * L_3')^{L_1'' * L_2'' + \tilde{L}_2'' * \tilde{L}_1''} =$$

$$= (L_1' * (L_2' + L_3'))^{L_1'' * L_2'' + \tilde{L}_2'' * \tilde{L}_1''};$$

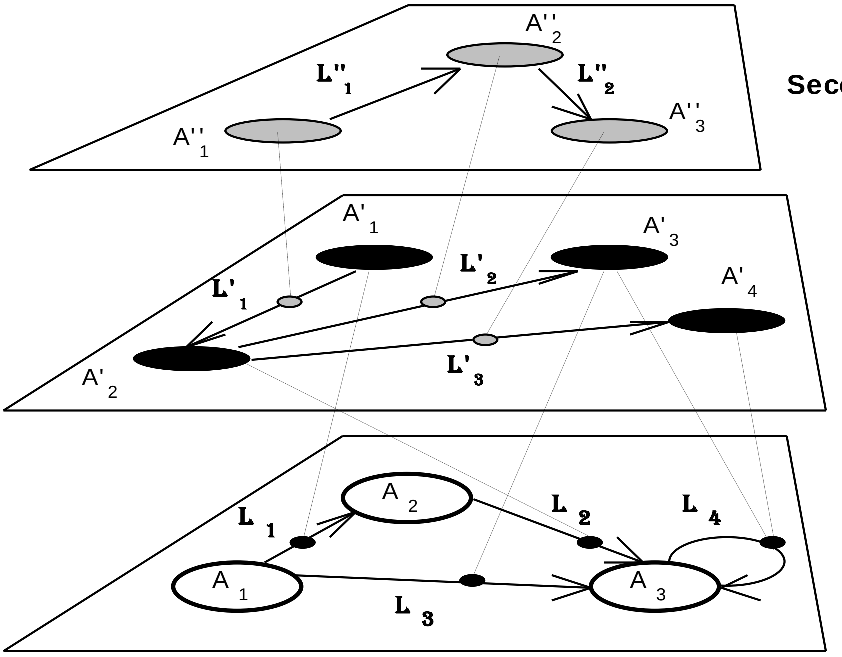
$$L_{A_2'} = (\tilde{L}_1')^{L_1'' * L_2''} + (L_2')^{\tilde{L}_1'' + L_2''} + (L_3')^{\tilde{L}_2'' * \tilde{L}_1''} =$$

$$= (\tilde{L}_1' + L_2' + L_3')^{L_1'' * L_2'' + \tilde{L}_2'' * \tilde{L}_1''};$$

$$L_{A_3'} = (\tilde{L}_2' * \tilde{L}_1')^{L_1'' * L_2'' + \tilde{L}_1''} + (\tilde{L}_2' * L_3')^{L_2'' + \tilde{L}_2'' * \tilde{L}_1''} =$$

$$= (\tilde{L}_2' * (\tilde{L}_1' + L_3'))^{L_1'' * L_2'' + \tilde{L}_2'' * \tilde{L}_1''};$$

Example of Decontextualization (4)



Now it is possible to derive the ***interpreted knowledge*** about the relation between $\boldsymbol{A_1}$ and $\boldsymbol{A_3}$ taking all contexts and metacontexts into account:

$$L_{A_1-A_3} = (L_1)^{L_{A_1}'} * (L_2)^{L_{A_2}'} + (L_3)^{L_{A_3}'} =$$

$$= (L_1 * L_2 + L_3)^{(L_1' * L_2' + L_1' * L_3' + \tilde{L}_2' * \tilde{L}_1' + \tilde{L}_2' * L_3')^{(L_1'' * L_2'' + \tilde{L}_2'' * \tilde{L}_1'')}} .$$

Reasoning with Contexts

Deriving unknown knowledge that is interpreted when the result of interpretation and the context of interpretation are known (contextualization).

This problem occurs when some knowledge has been interpreted in some context and we have all knowledge about this context and knowledge that is the result of interpretation.

For example, let us suppose that your colleague, whose context you know well, has described you a situation. You use knowledge about context of this person to interpret the "real" situation. Example is more complicated if several persons describe you the same situation. In this case, the context of the situation is the semantic sum over all personal contexts.

This second reasoning problem can be solved using the following equation:

$$L_x^{<\text{knowledge about context}>} = <\text{interpreted knowledge}>.$$

Reasoning with Contexts

Deriving unknown context of interpretation when the knowledge and its interpretation in this context are known (context recognition).

This problem occurs when we have knowledge that has been interpreted in some unknown context and we also know what is the result of interpretation.

For example let us supposed that someone sends you a message describing the situation that you know well. You compare your own knowledge with the knowledge you received. Usually you can derive your opinion about the sender of this letter. Knowledge about the source of the message, you derived, can be considered as certain context in which real situation has been interpreted and sometimes it can help you to recognize a source or at least his motivation to change the reality.

This third reasoning problem can be solved using the following equation:

$$< \text{knowledge} >^{L_X} = < \text{interpreted knowledge} >.$$

Reasoning with Contexts

Lifting (relative decontextualization).

This means deriving knowledge interpreted in some context if it is known how this knowledge was interpreted in another context.

This problem is solved by successive solution of the ***contextualization*** and ***decontextualization*** problems. Let $\boldsymbol{L_k}$ is the result of interpretation of some knowledge L_x in the context $\boldsymbol{L_m}$. The problem is to derive how this knowledge would be interpreted in the context $\boldsymbol{L_n}$. Thus we have the following procedure of lifting:

$$(L_x^{L_m} = L_k) \Leftrightarrow (L_x = L_k^{\overline{L}_m})[contextualization] \Leftrightarrow$$

$$\Leftrightarrow (L_x^{L_n} = L_k^{\overline{L}_m^{L_n}})[decontextualization]$$

Discussion

1. **How to interpret the concept of Metasemantic Network in the context of multiple experts problems ?**

2. **What in this case would mean the semantic interpretation operation ?**

3. **What in this case would mean all four reasoning problems with contexts ?**